

**American University of Beirut**  
**Department of Electrical and Computer Engineering**

**EECE 320 – Digital Systems Design**  
**Assignment 2**

16/10/2007

Suggested problems: Solve the drill problems 4.6-4.11 of chapter 4.

**Problem 1:** Simplify the Boolean expressions to a minimum number of literals:

- a)  $wxy' + wy'z' + wx'z'$
- b)  $ABC + A'B + ABC'$
- c)  $(BC' + A'D)(AB' + CD')$
- d)  $A'B(D' + C'D) + B(A + A'CD)$  (reduce to one literal)
- e)  $(A' + C)(A' + C')(A + B + C'D)$  (reduce to 4 literals)

**Problem 2:** Find the complement of the following functions

- a)  $AB(C'D + CD') + A'B'(C' + D)(C + D')$
- b)  $(x + y' + z)(x' + z')(x + y)$

**Problem 3:** Draw the following functions using OR gates and inverters only.

- a)  $F = x'y' + x'z + y'z$
- b)  $G = (y + z')(x + y)(y' + z)$

**Problem 4:** Repeat problem3 using AND gates and inverters only.

**Problem 5:** For the Boolean function F given in truth table, find the following:

w	x	y	z	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

- a) F as a Minterm list using  $\Sigma$  notation
- b) F as a Canonical sum
- c) F as a Maxterm list using  $\Pi$  notation
- d) F as a Canonical product
- e) The complement of F as sum of minterms
- f) Simplify F to a minimum number of literals

**Problem 6:** Convert the following expressions into 1) minimal sum of products 2) minimal product of sums and 3) sum of minterms. (Draw the corresponding truth table)

a)  $(AB+C)(B+C'D)$

b)  $x' + x(x+y')(y+z')$

# Assignment 2

## Solution

1) a)  $wxy' + wy'z' + wx'z' = w(xy' + x'z' + y'z') = w(xy' + x'z')$   
*By consensus*

b)  $ABC + A'B + ABC' = AB(C + C') + A'B = AB + A'B = B$

c)  $(BC' + A'D)(AB' + CD') = AB'BC' + AB'A'D + CD'BC' + CD'A'D$   
 $= 0$

d)  $A'B(D' + C'D) + B(A + A'CD) = B(A'D' + A'C'D + A + A'CD)$   
 $= B(A'D' + A + A'D(C' + C)) = B(A + A'(D' + D)) = B(A + A') = B$

e)  $(A' + C)(A' + C')(A + B + C'D) = (A' + CC')(A + B + C'D)$   
 $= A'(A + B + C'D) = A'B + A'C'D = A'(B + C'D)$

2) a)  $[AB(C'D + CD') + A'B'(C' + D)(C + D')]' =$

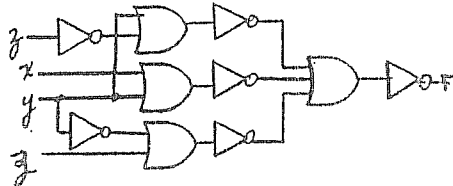
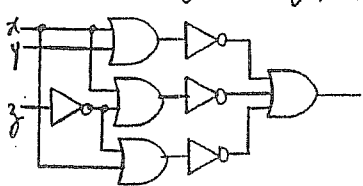
$[(A' + B') + (C + D')(C' + D)][(A + B) + CD' + C'D]$

b)  $(x + y' + z)(x' + z')(x + y) = x'y'z' + xz + x'y'$

3)

(a)  $F = x'y' + x'z + y'z$   
 $= (x+y)' + (x+z)' + (y+z)'$

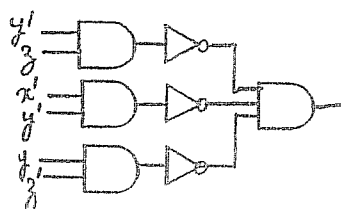
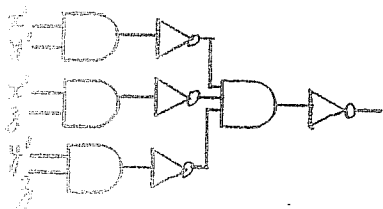
(b)  $F = (y+z')(x+y)(y'+z)$   
 $= [(y+z')' + (x+y)' + (y'+z)']'$



4)

(a)  $F = x'y' + x'z + y'z$   
 $= [(x'y')' + (x'z)' + (y'z)']'$

(b)  $F = (y+z')(x+y)(y'+z)$   
 $= (y'z)' + (x'y)' + (y'z)'$



Problem 5

$$a) F = \sum_{w,x,y,z} (1, 5, 6, 7, 9, 10, 11, 13, 14, 15)$$

$$b) F = w'x'y'z + w'x'y'z' + w'x'yz + w'x'yz' + wx'y'z + wx'y'z' + wx'yz + wx'yz' + wx'yz' + wx'yz' + wx'yz' + wx'yz'$$

$$c) F = \prod_{w,x,y,z} (0, 2, 3, 4, 8, 12)$$

$$d) F = (w+x+y+z)(w+x+y'+z)(w+x+y'+z')(w+x'+y+z)(w'+x+y+z)(w'+x'+y+z)$$

$$e) F' = \sum_{w,x,y,z} (0, 2, 3, 4, 8, 12) \quad (\text{Since minterms of } F' \text{ are the maxterms of } F)$$

$$f) y'z \left( \underbrace{w'x' + w'x + wx' + wx}_{\substack{w' \\ 1}} \right) + yz' \left( \underbrace{w'x + wx + wx'}_{\substack{x + w}} \right) + yz \left( \underbrace{w'x + wx + wx'}_{\substack{x + w}} \right) =$$

$$F = y'z + yz'(x+w) + yz(x+w)$$

$$F = y'z + y(x+w) \quad \dots \quad 5 \text{ literals.}$$

# Problem 6

$$\begin{aligned} a) (AB+c)(B+c'D) &= AB + A\cancel{B}c'D + BC + 0 \\ &= AB + BC \quad (\text{Sum of Products}) \\ &= B(A+c) \quad (\text{Product of Sums}) \end{aligned}$$

truth table

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$F = \sum_{A,B,C,D} (6, 7, 12, 13, 14, 15)$$

$$b) F = X' + \underbrace{X(X+Y')}(1)(Y+Z') = \underbrace{(X'+X)}_1 (X' + (X+Y')(Y+Z'))$$

$$F = X' + XY + Y'Z'$$

$$\text{since } X' + XY = X' + Y \quad \left( \underbrace{(X'+X)}_1 (X'+Y) = X' + Y \right)$$

~~and~~ Then

$$F = X' + \underbrace{Y + Y'Z'} = X' + Y + Z' \quad \begin{array}{l} \swarrow \text{SOP and} \\ \searrow \text{POS} \end{array}$$

$$F = \sum_{X,Y,Z} (0, 1, 2, 3, 4, 6, 7)$$

(since  $F = \text{Maxterm } 5$ )